

MATHEMATICS TODAY - OCT 09 - IIT-JEE PAPER-I
Answer key & Solutions

P H Y S I C S

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
d	b	c	c	a	b	b	c	d	ac	bc	ab	abcd	c	c
16														
a														

17. 0.76
 18. $100J$
 19. $A \rightarrow p;q;r$, $B \rightarrow p;q$, $C \rightarrow s$, $D \rightarrow q$
 20. $A \rightarrow p;r$, $B \rightarrow q;s$, $C \rightarrow p;r$, $D \rightarrow q;s$

C H E M I S T R Y

21	22	23	24	25	26	27	28	29	30	31	32	33	34	35
c	b	d	a	d	b	d	a	d	abc	ab	abc	abc	a	d
36														
c														

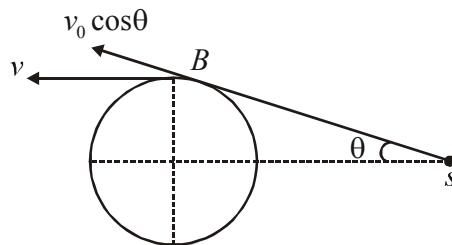
37. 100 pm
 39. $(A)-(q),(r);(B)-(q),(s); (C)-(p);(D)-(q),(s)$
 40. A-q, B-S, C-p, D-r

M A T H E M A T I C S

41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
d	d	b	c	a	d	a	c	a	cd	ad	abc	acd	a	b
56														
d														

57. 000001
 58. 000008
 59. $A \rightarrow r$, $B \rightarrow p$, $C \rightarrow s$, $D \rightarrow q$
 60. (A) $\rightarrow r$, (B) $\rightarrow p$, (C) $\rightarrow r$, (D) $\rightarrow q$

1.(d) The location of detector is shown



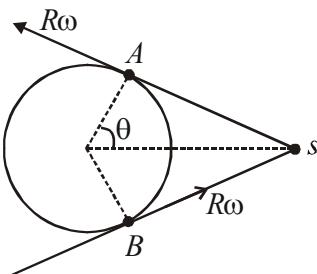
$$v_0 = R\omega \text{ (speed of detector)}$$

$$f_{ap} = \frac{v - v_0 \cos \theta}{v} \times f$$

$$\cos \theta = \frac{2R}{\sqrt{5}R}$$

$$f_{ap} = \frac{v - R\omega}{v} \times \frac{2}{\sqrt{5}} \times f$$

2.(b) A is corresponding to minimum frequency and B corresponds to maximum frequency



$$\cos \theta = \frac{R}{2R} = \frac{1}{2}, \theta = \frac{\pi}{3}$$

Let detector reaches at A in time t_0

$$\theta = \omega t_0, t_0 = \frac{\pi}{3\omega}$$

Let detector reaches at B in time t_0' from A.

$$2\pi - 2\theta = \omega \cdot t_0'$$

$$t_0' = \frac{4\pi}{3\omega} \quad \therefore \quad t_0 + t_0' = \frac{5\pi}{3\omega}$$

3.(c) Time interval $t_0' = \frac{4\pi}{3\omega}$

4.(c) Time constant $\tau = \frac{L}{R} = \frac{50mH}{10} = 5ms$

Growth equation in L-R circuit is

$$i = i_0(1 - e^{\frac{-R}{L}t})$$

$$\text{or } \frac{i_0}{2} = i_0(1 - e^{\frac{-R}{L}t}) \Rightarrow \frac{1}{2} = 1 - e^{\frac{-R}{L}t}$$

$$e^{\frac{-R}{L}t} = \frac{1}{2} \quad \text{or} \quad e^{\frac{R}{L}t} = 2$$

$$\frac{R}{L} \cdot t = \ln 2$$

$$t = \frac{L}{R} \cdot \ln 2 = 5 \times 10^{-3} \times 0.693 \Rightarrow t = 3.5ms$$

Choices (a), (b) and (d) are wrong.

5.(a) Current in $L - R$ circuit is given as $i = i_0(1 - e^{-t/\tau})$

where $i_0 = \frac{E}{R}$ and $\tau = \frac{L}{R}$

$$Q = \int_0^\tau i dt = i_0 \int_0^\tau (1 - e^{-t/\tau}) dt = i_0 \left[t - \frac{e^{-t/\tau}}{-1/\tau} \right]_0^\tau = \frac{i_0 \tau}{e}$$

Choices (b), (c) and (d) are wrong.

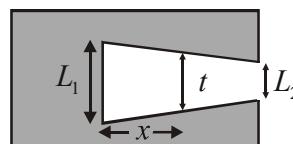
6.(b) $i = i_0 e^{-t/\tau}$

Here $i = \frac{i_0}{\eta}$ or $\frac{i_0}{\eta} = i_0 e^{-t_0/\tau} \Rightarrow \frac{1}{\eta} = e^{-t_0/\tau}$

or $\eta = e^{t_0/\tau} \Rightarrow \frac{t_0}{\tau} = \ln \eta \quad \text{or} \quad \tau = \frac{t_0}{\ln \eta}$

Choices (a), (c) and (d) are wrong.

- 7.(b)** From the theory mentioned in passage at the ends of cavity fringes will form, and as number of dark fringes is greater than the number of bright fringes so ends will be the location of dark fringes.



Thickness of the cavity at a distance x from the left end would be

$$t = L_2 + \frac{L_1 - L_2}{L} \times x$$

For left end,

$$2\mu L_1 = n\lambda_0 \quad [\text{For dark fringe}]$$

For right end,

$$2\mu L_2 = (n-7)\lambda_0 \quad \Rightarrow \quad L_1 - L_2 = \frac{7\lambda_0}{2\mu}$$

8.(c) For 4th dark fringe from left end

$$2\mu t = (n-3)\lambda_0$$

$$\Rightarrow 2\mu \left[L_2 + \frac{L_1 - L_2}{L} x \right] = 2\mu L_1 - 3\lambda_0$$

$$\Rightarrow x = \frac{4L}{7}$$

9.(d) 1st dark fringe is at left end, only.
So, 2nd dark fringe would be at

$$2\mu \left[L_2 + \frac{L_1 - L_2}{L} x \right]$$

$$= (n-1)\lambda_0 = 2\mu L_1 - \lambda_0 \Rightarrow x = \frac{6L}{7}$$

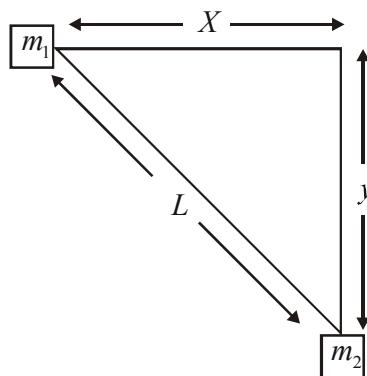
Sol.10: From momentum conservation

$$mV_0 = mV + 2mV$$

$$V = \frac{V_0}{3}$$

$$\text{Sol.11. } X_{cm} = \frac{m_1[-(L-y)] + m_2 \times 0}{m_1 + m_2}$$

$$y_{cm} = \frac{m_1 \times 0 + m_2(-y)}{m_1 + m_2}$$



Differentiating twice wrt time find we can find out acceleration.

Sol.12 Applying B.T. at point B and E . We get

$$P_0 + 0 + 0 = P_0 - \rho g h_1 + \frac{1}{2} \rho v^2$$

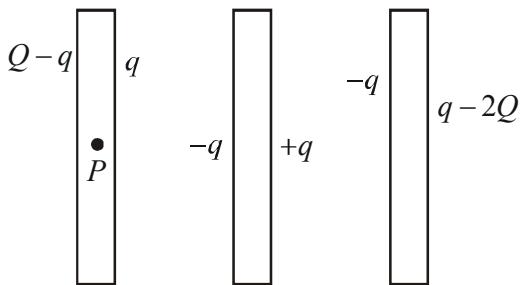
$$V = \sqrt{2gh_1} \text{ at } E.$$

Velocity at all the points outside the liquid has to be same from B.T.

From Pascal's law

$$P_D = P_0 - \rho g(h_1 + h_2)$$

Sol.13 Distribution of charges have been shown. The field at P



$$E = \frac{Q-q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{q}{2A\epsilon_0} + \frac{q}{2A\epsilon_0} - \frac{q-2Q}{2A\epsilon_0} = 0$$

$$q = \frac{3Q}{L}$$

we can find charge on different surfaces.

Sol.14: Velocity of A relative to B

$$V_{AB} = |\vec{V}_A - \vec{V}_B| = \sqrt{V^2 - V_1^2}$$

$$t = \frac{a}{V_{rel}} = \frac{a}{\sqrt{V^2 - V_1^2}}$$

$$\text{Sol.15 } F = -\frac{dU}{dr} = -2ar$$

$$\text{now } mvr = \frac{nh}{2\pi} \quad \dots \text{(i)}$$

$$\text{and } \frac{mv^2}{r} = 2ar \quad \dots \text{(ii)}$$

$$\text{Solving (i) and (ii), we get } r = \left(\frac{n^2 h^2}{8am\pi^2} \right)^{\frac{1}{4}}$$

Sol.16 length of connector at time $t = 2x$,

$$\therefore y = x^2, \quad x = \sqrt{y}, \quad l = 2\sqrt{y}$$

We know $V^2 = 2ay$, $V = \sqrt{2ay}$

So emf = BLV = $B \cdot 2\sqrt{y} \cdot \sqrt{2ay}$

$$e = 2By\sqrt{2a}$$

Sol.17 $t = \frac{A}{a} \sqrt{\frac{2}{g}} (\sqrt{h_1} - \sqrt{h_2})$

$$\text{Ratio of times} = \frac{\sqrt{30} - \sqrt{20}}{\sqrt{20} - \sqrt{10}} = \frac{5.47 - 4.47}{4.47 - 3.16} \\ = 0.76$$

Sol.18 $F = 6$, $\Delta W = +25J$, $\gamma = 1 + \frac{2}{f} = 1 + \frac{1}{3} = \frac{4}{3}$

From is Law

$$\Delta W = \Delta Q - \Delta U$$

$$\frac{\Delta W}{\Delta Q} = 1 - \frac{\Delta U}{\Delta Q} = 1 - \frac{nc_v d_T}{nc_p dT}$$

$$= 1 - \frac{C_v}{C_p} = 1 - \frac{1}{\gamma}$$

$$\frac{\Delta Q}{\Delta W} = \frac{\gamma}{\gamma - 1} = \frac{\frac{4}{3}}{\frac{4}{3} - 1} = 4$$

$$\Delta Q = 4 \times \Delta W = 100J$$

Ans.19 A → p;q;r , B → p;q , C → s, D → q

Sol.19 MNXYM is a cyclic process, $\Delta U = 0$

So all the supplied heat will do work.

NX is an isobaric process leading to the work, by the energy absorbed from external source.

Work done in NX is $+nRdT$ and in YM it is $-nRdT$.

In XY, there is drop in pressure at constant temperature, work done is $2.3026nRT \log\left(\frac{4}{2}\right)$

Ans.20 A → p;r , B → q;s , C → p;r, D → q;s

Sol.20 Bass strings have low fundamental frequency and larger wavelength. For having low frequency

the string has to be long according to expression $f \propto \frac{V}{L}$. For low f , v should be low, or string should be thick.

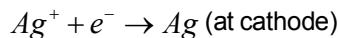
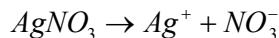
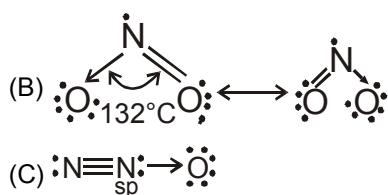
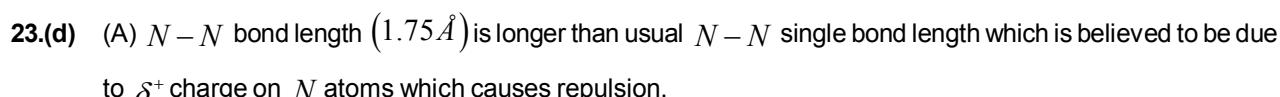
For treble strings also, the same explanation holds good.

C H E M I S T R Y

(reducing agent) (oxidising agent)

(C) Neutral to litmus as neutral oxide

(D) In the liquid state, N_2O_4 tends to ionise



108 g of silver is deposited by 96500 coulombs

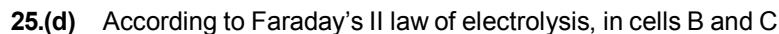
1.45 g silver is deposited by

$$\frac{96500 \times 1.45}{108} = 1295.6C$$

$$Q = C \times t$$

$$1295.6 = 1.5 \times t$$

$$t = \frac{1295.6}{1.5} = 863.7 \approx 864.0s$$



$$\frac{\text{Weight of Ag}}{\text{Weight of Cu}} = \frac{\text{Equivalent weight of Ag}}{\text{Equivalent weight of Cu}}$$

$$= \frac{1.45}{x} = \frac{108}{31.75}$$

$$x = \frac{31.75 \times 1.45}{108} = 0.426g$$



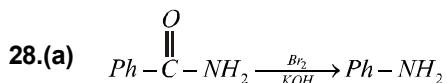
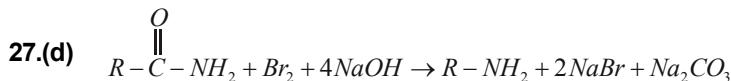
$$\frac{\text{Weight of Ag}}{\text{Weight of Zn}} = \frac{\text{Equivalent weight of Ag}}{\text{Equivalent weight of Zn}} = \frac{1.45}{x} = \frac{108}{32.75}$$

$$x = \frac{32.75 \times 1.45}{108} = 0.440 \text{ g}$$

Weight of zinc deposited = 0.440g

Sol. 27, 28, 29

(Balanced reaction of Hoffmann bromamide reaction)



29.(d) N - substituted amide will not undergo Hoffmann bromamide reaction because formation of isocyanate is not possible.

30.(a,b,c).

Boron nitride has layered lattice structure. Each layer consists of a hexagonal arrangement of B and N atoms and this structure resembles that of graphite.

31.(a,b). $3\text{O}_2 \rightarrow 2\text{O}_3$

$$(90-9) = 81 \quad \frac{2}{3} \times 9 = 6$$

Before passing through turpentine oil, the volume of gases = $81 + 6 = 87 \text{ ml} = V_1$

After passing through turpentine oil, the volume of gases left = $81 \text{ ml} = V_2$.

32. (a,b,c).

33. (a), (b), (c) activated benzene rings due to lone pair of electrons on attaching atom.

34.(a). $K_a = 1.85 \times 10^{-5} = \frac{\alpha^2 C}{(1-\alpha)} \approx \frac{\alpha^2 C^2}{C}$

$$\therefore (\alpha C) = [H^+] = [1.85 \times 10^{-5} \times C]^{\frac{1}{2}}$$

In the I case,

$$[H^+] = [1.85 \times 10^{-5} \times 0.1]^{\frac{1}{2}} = 1.36 \times 10^{-3} M$$

$$pH = -\log [1.36 \times 10^{-3}] = 3 - \log 1.36 = 3 - 0.1335 = 2.8665$$

In the II case,

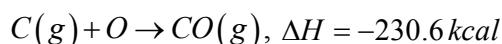
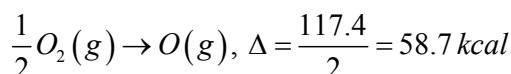
$$[H^+] = [1.85 \times 10^{-5} \times 0.01]^{\frac{1}{2}} = (18.5 \times 10^{-8})^{\frac{1}{2}} = 4.30 \times 10^{-4} M$$

$$\therefore pH = -\log (4.30 \times 10^{-4}) = 4 - \log 4.3 = 4 - 0.6335 = 3.3665$$

$$\therefore \text{change in } pH = 3.3665 - 2.8665 = 0.5$$

35.(d) From the first two equations we get $C(s) + \frac{1}{2}\text{O}_2(g) \rightarrow CO(g)$, $\Delta H = -26.9 \text{ kcal/mol}$

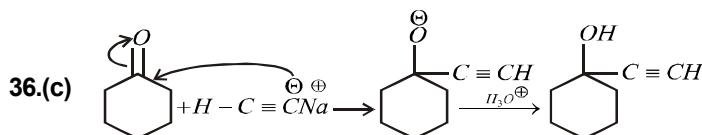
We now consider $C(s) \rightarrow C(g)$, $\Delta H_1 \text{ kcal}$



Adding we get,

$$\Delta H_1 + 58.7 - 230.6 = -26.9$$

$$\therefore \Delta H_1 = (-26.9 + 230.6 - 58.7) \text{ kcal/mol} = 145 \text{ kcal/mol}$$



- 37.** Effective molality of solution = 1
Hence, no. of moles of ionic solid in given cube = 0.5

$$\text{so, no. of formula units in given cube} = \frac{1}{2} \times 6 \times 10^{23}$$

$$\text{no. of unit cells} = \frac{1}{4} \times \frac{1}{2} \times 6 \times 10^{23} = 7.5 \times 10^{22}$$

$$\text{no. of unit cells along one edge of cube} = \sqrt[3]{75} \times 10^7 = 4.22 \times 10^7$$

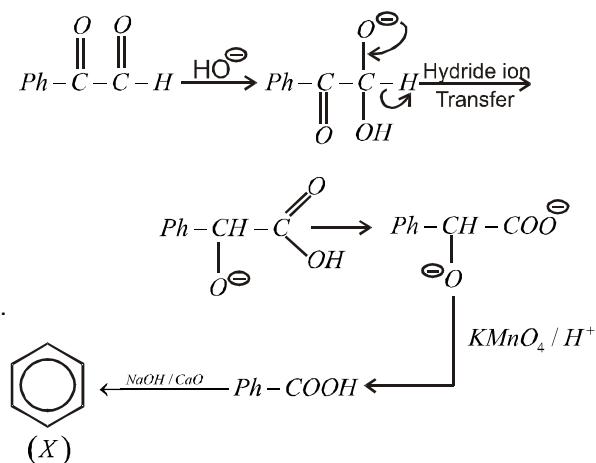
$$\text{If edge length of unit cell} = a = \frac{25.32 \times 10^{-3}}{4.22 \times 10^7}$$

$$m = 600 \times 10^{-12} \text{ m} = 600 \text{ pm}$$

$$\text{for } NaCl \text{ type unit cell, } a = 2(r^+ + r^-)$$

$$\text{So } r^+ = 100 \text{ pm}$$

38.



39. $(A)-(q),(r);(B)-(q),(s); (C)-(p);(D)-(q),(s)$

(A) $[Mn(CN)_6]^{3-}$: paramagnetic, d^2sp^3

(B) $[MnCl_4]^{2-}$: paramagnetic sp^3

(C) $[Ni(CN)_4]^{2-}$: diamagnetic, dsp^2

(D) $[NiCl_4]^{2-}$: paramagnetic, sp^3

40. A-q, B-S, C-p, D-r

M A T H E M A T I C S

Sol. **41, 42, 43**

Since equations $ax^3 + bx^2 + cx + d = 0$ and $x^3 + cx^2 + bx + 3 = 0$ have same roots therefore

$$\frac{a}{1} = \frac{b}{c} = \frac{c}{b} = \frac{d}{3} \Rightarrow b^2 = c^2 \text{ and } d = 3a \Rightarrow b = -c \text{ and } d = 3a$$

also since

$$d = a + 1 \Rightarrow a = 1/2 \quad \text{and} \quad d = 3/2$$

further $b - c = 10 \Rightarrow b = 5$ and $c = -5$

∴

41. $a + b + c + d = 2$

$$42. \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & 5 & -5 \\ 5 & -5 & \frac{1}{2} \\ -5 & \frac{1}{2} & 5 \end{vmatrix}$$

$$C_1 \rightarrow C_1 + C_2 + C_3$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 5 & -5 \\ 1 & -5 & \frac{1}{2} \\ 1 & \frac{1}{2} & 5 \end{vmatrix} \neq 0$$

∴ A is an invertible matrix.

Since $AA^T \neq I$ also $AdjA \neq A^2$

therefore correct answer is (d)

43. $f(x) = \frac{1}{2}x^3 + 5x^2 - 5x + \frac{3}{2}$

$$\Rightarrow 2f(x) = x^3 + 10x^2 - 10x + 3 \Rightarrow 2f(x) + 35x = x^3 + 10x^2 + 25x + 3 = y$$

$$\Rightarrow \frac{dy}{dx} = 3x^2 + 20x + 25 = (x+5)(3x+5)$$

Since $f(-5) = 3 > 0$ and $f(-5/3) < 0$

$\therefore y=0$ has three real roots.

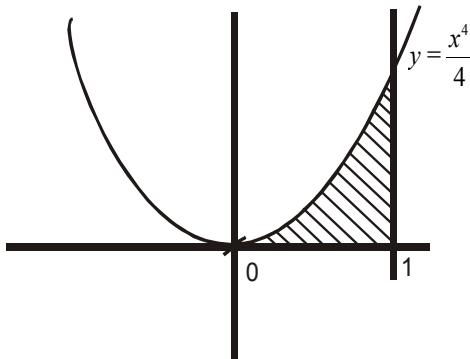
Sol. 44, 45, 46

$$\frac{dy}{dx} = kx^3 \Rightarrow y = \frac{kx^4}{4} + C$$

Since A lies on curve $\therefore C = 0$ Hence curve is $y = \frac{kx^4}{4}$

44. Since $B(\sqrt{2}, 1)$ lies on curve $\therefore k = 1 \Rightarrow$ eq of curve is $y = \frac{x^4}{4}$

$$\therefore \text{Required area} = \int_0^1 \frac{x^4}{4} dx = \left(\frac{x^5}{20} \right)_0^1 = \frac{1}{20}$$



45. $\lim_{x \rightarrow \sqrt{2}} \left(\frac{kx^4}{4} \right)^{\frac{1}{x^4-4}} = e^{1/4} \Rightarrow k = 1$

46. Since $B(1, 1)$ lies on the curve $\therefore k = 4 \Rightarrow$ curve is $y = x^4$

let (t, t^4) be any point on the curve

$$\therefore \text{eq of tangent at } (t, t^4) \text{ is } y - t^4 = 4t^3(x - t)$$

let it passes through $(2, 0)$

$$-t^4 = 4t^3(2 - t) \Rightarrow 3t^4 - 8t^3 = 0$$

$$\Rightarrow t^3 \left(t - \frac{8}{3} \right) = 0 \quad \Rightarrow t = 0, t = \frac{8}{3} \quad \therefore R \equiv (0,0) \quad \& \quad S \equiv \left(\frac{8}{3}, \frac{4096}{81} \right)$$

∴ Area of ΔPBR

$$= \frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1$$

Area of Δ PBS

$$\begin{aligned} &= \frac{1}{2} \begin{vmatrix} 2 & 0 & 1 \\ 1 & 1 & 1 \\ \frac{8}{3} & \frac{4096}{81} & 1 \end{vmatrix} \\ &= \frac{1}{2} \left| 2 \left(1 - \frac{4096}{81} \right) + 1 \left(\frac{4096}{81} - \frac{8}{3} \right) \right| \\ &= \frac{1}{2} \left| -\frac{4015 \times 2}{81} + \frac{4096 - 216}{81} \right| \\ &= \frac{1}{2} \left\{ \frac{4150}{81} \right\} \\ &= \frac{2075}{81} \end{aligned}$$

$$\therefore \text{Area } \Delta PBR : \text{Area } \Delta PBS = \frac{81}{2075}$$

Sol. **47, 48, 49**

Let the prob. that head not occurs is $q \Rightarrow q = 1 - p$

$$47. \quad \beta = P(\overline{AB} + \overline{A}\overline{B}\overline{C}AB + \overline{A}\overline{B}\overline{C}A\overline{B}\overline{C}AB + \dots \infty)$$

$$= qp + q^4 p + q^7 p + \dots \infty = \frac{qp}{1-q^3} = q\alpha = (1-p)\alpha$$

$$48. \quad \therefore \alpha = P(A + \overline{A}\overline{B}\overline{C}A\overline{B}\overline{C}A + \dots \infty) = p + q^3 p + q^6 p + \dots \infty = \frac{p}{1-q^3} = \frac{p}{1-(1-p)^3}$$

$$49. \quad \gamma = 1 - (\alpha + \beta) = 1 - (\alpha + (1-p)\alpha) = 1 - \alpha(2-p) = 1 - \frac{p(2-p)}{1-q^3}$$

$$\text{if } p = \frac{1}{2} \Rightarrow \gamma = \frac{1}{7}$$

$$50.(c,d) \quad \lim_{x \rightarrow 0^+} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} = \lim_{x \rightarrow 0^+} \frac{1 - e^{-2/x}}{1 + e^{-2/x}} = 1 \text{ and}$$

$$\lim_{x \rightarrow 0^-} \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}} = \lim_{x \rightarrow 0^-} \frac{e^{2/x} - 1}{e^{2/x} + 1} = -1$$

Hence $\lim_{x \rightarrow 0} f(x)$ exists if $g'(0) = 0$

If $g(x) = ax + b, a \neq 0$ then $\lim_{x \rightarrow 0^+} f(x) = a$

and $\lim_{x \rightarrow a^-} f(x) = -a$ hence $\lim_{x \rightarrow 0} f(x)$ exists if $g(x) = x^2$

or $g(x) = x^3 h(x)$ where $h(x)$ is a polynomial.

51.(a,d)

Using $\vec{a} + 2\vec{b} + 3\vec{c} = \vec{0}$ we can put $\vec{a} = -(2\vec{b} + 3\vec{c})$ or $2\vec{b} = -(\vec{a} + 3\vec{c})$ or $3\vec{c} = -(\vec{a} + 2\vec{b})$ in $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ to get its value in terms of \vec{b}, \vec{c} or \vec{a}, \vec{c} or \vec{a}, \vec{b} e.g. $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = 6(\vec{b} \times \vec{c})$

52. (a,b,c).

Any point on the line $x + y = 1$ can be taken $(t, 1-t)$. Equation of the chord, with this as mid point is $y(1-t) - 2a(x+t) = (1-t)^2 - 4at$, it passes through $(a, 2a)$. So, $t^2 - 2t + 2a^2 - 2a + 1 = 0$ this should have two distinct real roots so discriminant > 0 , we get $a^2 - a < 0 \Rightarrow 0 < a < 1$, therefore, latus rectum < 4 .

53. (a,c,d).

$$A : \left(\frac{-y + x \frac{dy}{dx}}{\frac{dy}{dx}}, 0 \right) \quad B : (x, 0)$$

$$\text{Now, } 2x \frac{dy}{dx} = -y + x \frac{dy}{dx} \Rightarrow x \frac{dy}{dx} = -y \Rightarrow \frac{dx}{x} = -\frac{dy}{y} \Rightarrow xy = c$$

$$\text{since } f(1) = 1$$

$$\Rightarrow c = 1 \quad \therefore \text{ curve is } xy = 1$$

$$\text{54.(a)} \quad \frac{dy}{dx} = x^2 - 2x \Rightarrow y = \frac{x^3}{3} - x^2 + C, \text{ now since it passes through (2,0), therefore,}$$

$$y = \frac{x^3}{3} - x^2 + \frac{4}{3}$$

$$\text{At the point of maximum ordinate since } \frac{dy}{dx} = 0$$

$$\Rightarrow x^2 - 2x = 0 \quad \Rightarrow x = 0 \text{ or } x = 2$$

$$\text{therefore required point is } \left(0, \frac{4}{3} \right)$$

55.(b) The graphs of $y = \sin \pi x$ and $y = |\ln|x|$ cut at 4 points in $x \in [-e, e], x \neq 0$. And for $|x| > e, |\ln|x|| > 1$ there is no solution for $\sin \pi x = |\ln|x||$. Therefore, number of solutions is 4.

56.(d) Differentiating both the sides of (1) we get

$$\begin{vmatrix} 1 & 5 & 0 \\ x^2 - 1 & x - 1 & 8 \\ 2x & 3x & 0 \end{vmatrix} + \begin{vmatrix} x - 1 & 5x & 7 \\ 2x & 1 & 0 \\ 2x & 3x & 0 \end{vmatrix} + \begin{vmatrix} x - 1 & 5x & 7 \\ x^2 - 1 & x - 1 & 8 \\ 2 & 3 & 0 \end{vmatrix} = 3ax^2 + 2bx + c$$

putting $x = 0$ we get

$$c = \begin{vmatrix} -1 & 0 & 7 \\ -1 & -1 & 8 \\ 2 & 3 & 0 \end{vmatrix} + \begin{vmatrix} -1 & 0 & 7 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} + \begin{vmatrix} 1 & 5 & 0 \\ -1 & -1 & 8 \\ 0 & 0 & 0 \end{vmatrix} = 17$$

57. Answer = 000001

$$f(xy) = \frac{f(x)}{y} + \frac{f(y)}{x}$$

Put $x = 1 \& y = 1 \Rightarrow f(1) = 0$.

$$\therefore f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = -\frac{f(x)}{x} + \frac{A}{x^2}$$

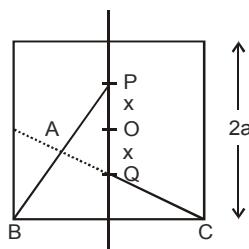
$$\text{Where, } A = \lim_{h \rightarrow 0} \frac{f\left(1 + \frac{h}{x}\right)}{\frac{h}{x}} \quad \therefore \quad \frac{d}{dx} xf(x) = \frac{A}{x}$$

$\therefore xf(x) = A \ln x + \ln c, \text{ put } x = 1 \Rightarrow c = 1$

$$\therefore xf(x) = A \ln x, \text{ Now } A = 1 \left(\because f(e) = \frac{1}{e} \right) \quad \therefore f(x) = \frac{\ln x}{x}$$

$$\therefore \text{Maximum of } f(x) = \frac{\ln e}{e} = \frac{1}{e}$$

58. Answer 000008



$$\tan B = \frac{a+x}{a}, \tan C = \frac{a-x}{a}$$

$$\begin{aligned} \text{Now, } & \tan A + \tan B + \tan C \\ &= \tan A \cdot \tan B \cdot \tan C \\ \Rightarrow & \tan A + 2 = \tan A \cdot \frac{a^2 - x^2}{a^2} \end{aligned}$$

$$\tan A \left(\frac{a^2 - x^2}{a^2} - 1 \right) = 2$$

$$\tan A = \frac{2a^2}{-x^2}$$

$$\tan A(\tan B - \tan C)^2 = \frac{2a^2}{-x^2} \frac{4x^2}{a^2} = -8 \quad \Rightarrow \quad \tan A(\tan B - \tan C)^2 + 16 = 8$$

59. $A \rightarrow r, B \rightarrow p, C \rightarrow s, D \rightarrow q$

(A) In the expansion of $\left(2 + \frac{x}{3} \right)^n$

$$T_{r+1} = {}^n C_r 2^{n-r} \left(\frac{x}{3} \right)^r \quad \therefore \text{ Coefficient of } x^7 = \text{coeff. of } x^8$$

$$\Rightarrow \frac{{}^n C_7 2^{n-7}}{3^7} = \frac{{}^n C_8 2^{n-8}}{3^8} \quad \Rightarrow \quad {}^n C_8 = 6 {}^n C_7$$

$$\Rightarrow \frac{\underline{n}}{\underline{n-8}\underline{8}} = 6 \frac{\underline{n}}{\underline{n-7}\underline{7}} \quad \Rightarrow \quad \frac{1}{8} = \frac{6}{n-7}$$

$$\Rightarrow n-7 = 48 \quad \Rightarrow \quad n = 55$$

$$\therefore A \rightarrow r$$

(B) In the expansion $\left(x - \frac{1}{ax^2} \right)^{10}$

$$T_{r+1} = (-1)^r {}^{10} C_r x^{10-r} \frac{1}{(ax^2)^r} \quad \therefore \quad 10 - 3r = 1$$

$$\Rightarrow 3r = 9 \quad \Rightarrow \quad r = 3$$

$$\text{Coeff. of } x = -\frac{{}^{10} C_3}{a^3} \Rightarrow -\frac{{}^{10} C_3}{a^3} = -15$$

$$\Rightarrow a^3 = 8 \quad \Rightarrow \quad a = 2 \quad \therefore \quad B \rightarrow p$$

$$\begin{aligned} (C) \quad 2^{2003} &= 2^3 (2^4)^{500} \\ &= 2^3 (17-1)^{500} \end{aligned}$$

$$= 2^3 \left[{}^{500}C_0 (17)^{500} - {}^{500}C_1 (17)^{499} + \dots + {}^{500}C_{500} \right]$$

\therefore when 2^{2003} is divided by 17 remainder = 8

$\therefore C \rightarrow s$

(D) $3^{400} = (3^2)^{200}$

$$= (10-1)^{200}$$

$$= {}^{200}C_0 (10)^{200} - {}^{200}C_1 (10)^{199} + \dots - {}^{200}C_{199} (10) + {}^{200}C_{200}$$

Clearly last two digits of the entire sum remains same : as the last two digits of the sum last three terms of above expansion = last two digits of

$$1990000 - 2000 + 1 = 01$$

$\therefore D \rightarrow q$

60. (A) $\rightarrow r$, (B) $\rightarrow p$, (C) $\rightarrow r$, (D) $\rightarrow q$

(A) The equation reduces to $z^5(1+z+z^2+\dots+z^5)=0$

$$\Rightarrow z^6 = 1 \quad \Rightarrow z = e^{i\frac{2\pi r}{6}} \quad r=1,2,\dots,5$$

\therefore The max side of the pentagon is $|z_5 - z_1| = 2 \sin \frac{\pi}{3} = \sqrt{3}$

(B) let the equation be $k(x-d)(x-p) = 0$

$k(\alpha-1)(\beta-1)$ is prime

$$\Rightarrow k=1 \text{ & } \alpha-1=1, \beta-1=2$$

$$\Rightarrow \alpha=2 \text{ & } \beta=3$$

\Rightarrow sum of the roots = 5

(C) $\|z-w| - |z-w^2|\| \leq |w^2-w|$

$$\therefore \|z_1| - |z_2\| \leq |z_1 - z_2| \leq \sqrt{3}$$

Since $|z| = \sqrt{3}$ Hence equality can hold good

$$\therefore y = \sqrt{3}$$

(D) $\sin^{-1} x + \tan^{-1} x + \sin^{-1} \sin x = 2k - 1$

The range of $\sin^{-1} x + \tan^{-1} x + \sin^{-1} \sin x$ is

$$\left[\frac{-3\pi}{4} - 1, \frac{3\pi}{4} + 1 \right]$$

$$\Rightarrow \frac{-3\pi}{4} - 1 \leq 2K - 1 \leq \frac{3\pi}{4} + 1 \quad \Rightarrow \quad \frac{-3\pi}{8} \leq K \leq \frac{3\pi}{8} + 1$$

Hence integral values K are -1, 0, 1, 2

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